B.A./B.Sc. 6th Semester (Honours) Examination, 2023 (CBCS)

Subject : Mathematics

Course : BMH6DSE43

(Mechanics II)

Time: 3 Hours **Full Marks: 60**

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Notation and symbols have their usual meaning.

1. Answer *any ten* questions: 2×10=20

(a) Obtain the Lagrangian of a simple pendulum.

(b) Define a conservative field of force. Give an example. 1+1

(c) Find the equation of free surface when a fluid is in equilibrium under the action of gravity only.

(d) Show that the distance between two points remains invariant under Galilean transformation.

(e) Define Holonomic and Non-Holonomic constraints.

(f) State Archimedes principle for a floating body.

(g) Write down the necessary and sufficient condition for equilibrium of a fluid under the action of external forces.

(h) If the forces per unit mass at (x, y, z) parallel to the axes are $y(a-z), x(a-z), xy$, then examine whether the force field is in equilibrium or not.

(i) If a parallelogram be immersed in any manner in a homogeneous liquid, then prove that the sum of the pressures at the extremities of each diagonal is the same.

(j) What is an adiabatic change of state?

(k) Explain briefly the term "Convective Equilibrium".

(l) Find the work done in compressing a gas from volume V to volume U isothermally.

(m) Write down the stress matrix at a point in an ideal fluid with proper explanation of symbol.

(n) What is an isothermal process? Give example. 1+1

(o) Give the interpretation of D'Alembert's principle.

2. Answer *any four* questions: 5×4=20

(a) ABC is a triangular lamina with the side AB in the surface of a heavy homogeneous liquid. A point D is taken in AC, such that the thrusts on the areas ABD and DBC are equal. Find the ratio AD : AC.

(b) Deduce the relation, $\frac{T}{T_0} = 1 - \frac{\gamma-1}{\gamma}\frac{z}{H}$, assuming gravity to be constant.

(c) A particle of mass m moving in a central force field under inverse square law. Find its P.E. and K.E. Also obtain Lagrange's equation of motion. 1+1+3

(d) A small uniform circular tube, whose plane is vertical contains equal quantities of fluids whose densities are ρ and $\sigma(\rho > \sigma)$, and do not mix. If they together fill half of the tube, show that the radius passing through the common surface makes with the vertical an angle $\tan^{-1}\left(\frac{\rho-\sigma}{\rho+\sigma}\right)$.

(e) Obtain differential equation for curves of equipressure and equidensity.

(f) A hollow weightless hemisphere with a plane base is filled with water and hung by means of a string, one end of which is attached to a point of the rim on its base. Find the inclination to the horizontal of the resultant thrust on its curved surface.

3. Answer *any two* questions: 10×2=20

(a) (i) A given volume V of liquid is acted upon by forces $-\frac{\mu x}{a^2}, -\frac{\mu y}{b^2}, -\frac{\mu z}{c^2}$ ($\mu > 0$, a constant). Find the equation of the free surface.

(ii) A semi-circular lamina of radius a is immersed in a liquid with diameter in the surface. Find the depth of the centre of pressure. 5+5

(b) The Lagrangian L for the motion of a particle of unit mass is

$$L = \frac{1}{2}(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) - V + A\dot{x} + B\dot{y} + C\dot{z}$$

where each of V, A, B, C is a given function of (x, y, z). Show that the equations of motion can be written in the form

$$\ddot{\vec{r}} = -\vec{\nabla}V + \dot{\vec{r}} \times curl\vec{s}$$

where $\vec{r} = (x, y, z), \vec{s} = (A, B, C)$.

(c) (i) Define Galilean transformation.

(ii) Show that acceleration remains invariant under Galilean transformation.

(iii) The stress tensor at a point continuum is given by,

$$(\tau_{ij}) = \begin{pmatrix} 3 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{pmatrix}.$$

Determine the principal stresses and the corresponding principal directions. 2+2+6

(d) (i) For a scleronomic dynamical system, show that the kinetic energy is a homogeneous quadratic function of generalized velocities.

(ii) A particle of mass m is projected in space with velocity v_0 at an angle α to the horizontal. Write the Lagrangian for the motion of the projectile and the equation of motion of the system. 6+(2+2)